‘Science’ of Monetary Policy: CGG (1999)

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Model Structure

- We have 3 equations
  (1) NKPC as the AS curve
  (2) Euler equation as the Expectational IS curve
  (3) Money Market clearing rule. [Ignore the monetary authority's objective function for now.]
  that characterize the behavior/dynamics of the economy.
- Interest rate as the Instrument of Monetary Policy in the spirit of Taylor.
- Hence 3 (generic) variables: $\{p_t\}$ (thus inflation), $\{x_t\}$ and $\{m_t\}$.
- But m_t does NOT appear in the IS equation or NKPC.
- $\Rightarrow$ For analytical purposes, we can ignore the money market equation.

Model Structure Cont.

- It considers optimal monetary policy. $\boxed{Note:}$ (i) The objective function has two variables $\{\pi_t\}$ and $\{x_t\}$. (ii) The NKPC has exactly the same two variables. (Ignore whether the variable is expressed as it is or in its expectation form.) It does not contain i_t.
- $\Rightarrow$ The Problem reduces to:

Maximize Objective Function, i.e. Minimize Loss Function, s.t. NKPC equ

- Solutions spell optimal times paths of inflation and output gap.
- Knowing this, we solve i_t from the expectational IS equation. Thus the IS equation implements the optimal policy through adjustment in the interest rate. Hence it is an implementation equation.
- Overall nature of the solution:
  (1) Constrained Optimization Problem: solves $\{\pi_t\}$ and $\{x_t\}$.
  (2) Given these, the expectational IS equation solves $\{i_t\}$
  (3) Given $\{\pi_t\}$, $\{x_t\}$ and $\{i_t\}$, solve $\{m_t\}$.
- Hence, cost push shock which shifts the NKPC is typically the focus.

Two Policy Implications

- Implication A: Any money or financial market shocks should be accommodated by changes in money supply. Interest rate, inflation or output shouldn't change .
- Implication B: Product market shocks affect the IS curve only. Hence any such shock is fully offset by interest rate adjustment, which is turn is accommodated by commensurate change in money stock. There is no change in the inflation rate or output.

NKPC and Expectational IS Curve

- Assume that $I_t = G_t = 0$, which is slightly more restrictive than CGG (1999), who assume $G_t > 0$; see their footnote #11. Then $C_t = Y_t$.
- Recall the NKPC equation: $\pi_t = \beta_t \pi_{t+1} + \alpha(p^* - p_t)$.
- We wrote $p^* - p_t = -(w_t - p^*) + w_t - p_t$. Recall the labor supply decision rule: $w_t - p_t = \sigma c_t + \Phi n_t$. But $c_t = y_t$. From technology $n_t = y_t$.
- $\Rightarrow w_t - p_t = (\sigma + \phi) y_t$; Similarly, $w_t - p_t^* = (\sigma + \phi) y_t^*$
- $\Rightarrow p^* - p_t = (\sigma + \Phi) x_t$.
- Compared to the NKPC derived earlier on the basis of Blanchard-Kiyotaki model, we have the additional term σ only; no other change.
- Because $c_t = y_t$, the expectational IS curve is the same.

Equations and Objective Function

- Objective Function: Minimize

$$\frac{1}{2}E_t\sum_{k=0}^{\infty}\beta^k\left[\lambda(\pi_{t+k}^2+x_{t+k}-x^*)^2\right] \quad (1)$$

- NKPC: $\pi_t = \beta E_t\pi_{t+1} + \kappa x_t + e_t,\ e_{t+1} = \rho^e e_t + \epsilon_t^e$. (2)
- Expectational IS Equation (The Implementation Equation):

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}(i_t - E_t\pi_{t+1}) + u_t, \quad u_{t+1} = \rho^u u_t + \epsilon_t^u. \quad (3)$$

- The model is closed.

Optimal Inflation and Output Gap under Full Commitment

- Implication C: The solution paths of x_t and π_t, given by the problem of minimizing (1) subject to (2) are time-inconsistent.

$$L = \frac{1}{2} E_t \sum_{k=0}^{\infty} \left\{ \beta^k \left[\lambda (x_{t+k} - x^*)^2 + \pi_{t+k}^2 \right] \right.$$
$$\left. + \tilde{\phi}_{t+k} (\pi_{t+k} - \kappa x_{t+k} - \beta \pi_{t+k+1} - e_{t+k}) \right\}.$$

- Eliminating the multipliers, FOC's are:

$$E_t \pi_{t+k} = \frac{\lambda (E_t x_{t+k-1} - E_t x_{t+k})}{\kappa}, \quad k \geq 1$$
$$\pi_t = -\frac{\lambda (x_t - x^*)}{\kappa}.$$

- The 2nd condition links 1-1 current inflation to current output gap. But from next period onwards, it links the would-be inflation to the output gap difference in the two consecutive period. When the next period become current, the rule becomes different.

Time Inconsistency Explained

- It is like when you optimize as of today what your life plan should be about eating and exercise today and for future dates, you always say that more exercise is good in the 'long run' but your current allocation of eating and exercise is different.
- Why? It is because the current period's inflation affects the current period's NKPC constraint facing the economy, whereas the next period's inflation affects that constraint for the next two periods.
- This follows from the forward looking nature of the NKPC. It wouldn't arise if we had the traditional Phillips curve.
- A Resolution: By commitment is meant a *timeless perspective* and completely ignoring 2nd FOC. It means the 1st FOC.
- Discretion means the 2nd FOC, ignoring the 1st FOC.

Timeless and Policy-Rule Commitments

- Timeless Commitment System:

$$x_t = a_x x_{t-1} + b_x e_t; \quad \pi_t = k_p x_{t-1} + k_e e_t.$$

- 1st order stochastic different equations.
- Implication D: Output gap and inflation have inertia, empirically observed.
- Drawback Complex.
- Second-best Policy Rule: Suppose $\pi_t = me_t$. Solve optimal m.
- Implication E: If optimal m is followed, there is no inertia.

Discretion: More Realistic Central Bank Behavior

- $\boxed{\pi_t = -\frac{\lambda(x_t - x^*)}{\kappa}}$. **Optimal Feedback Rule**
- Substitute the FOC into NKPC and eliminate π_t and ${}_t\pi_{t+1}$. We get

$$\left(1 + \frac{\kappa^2}{\lambda}\right) x_t = (1-\beta)x^* + \beta E_t x_{t+1} - \frac{\kappa}{\lambda} e_t.$$

We try $x_t = \gamma_0 + \delta e_t$. This leads to

$$x_t = \frac{(1-\beta)x^*}{1-\beta+\kappa^2/\lambda} - \frac{\kappa}{\lambda+\kappa^2} e_t; \quad \pi_t = \underbrace{\frac{\kappa x^*}{1-\beta+\kappa^2/\lambda}}_{\text{targeted inflation}} + \frac{\lambda}{\lambda+\kappa^2} e_t.$$

- Observe: inflationary bias, $\Leftarrow y^e - y^f \equiv x^* > 0$, while the objective is to minimize variation around y^e.
- Implication F: Optimal Feedback Policy package is of 'lean against the wind' kind.
- One solution: Commitment. May not be practical.
- Another solution due to Rogoff: Appoint a conservative banker.

Response to Shocks

$$x_t = \frac{(1-\beta)x^*}{1-\beta+\kappa^2/\lambda} - \frac{\kappa}{\lambda+\kappa^2}e_t; \quad \pi_t = \frac{\kappa x^*}{1-\beta+\kappa^2/\lambda} + \frac{\lambda}{\lambda+\kappa^2}e_t.$$

- $\partial x_t/\partial e_t < 0$, $\partial \pi_t/\partial e_t > 0$. By using the expectational IS relation, i_t should increase.
- Implication G: Demand shocks should be offset, cost shocks to be accommodated.
- Limitation: No inflation or output gap inertia, which is empirically observed, except exogenously if the cost push shock is autocorrelated.

Implied Taylor's Rule

Substituting the solutions of x_t and π_t into the IS equation,

$$
\begin{aligned}
i^t &= \frac{(1-\rho^e)\kappa\sigma + \lambda\rho^e}{\lambda+\kappa^2} e_t \\
&= BE_t\pi_{t+1}, \text{ where } B \equiv 1 + \frac{(1-\rho^e)\kappa\sigma}{\lambda\rho^e} > 0
\end{aligned}
$$

Endogenous Persistence

- Let $x_t = \frac{1}{\sigma}(i_t - E_t\pi_{t+1}) + \theta x_{t-1} + (1-\theta)E_t x_{t+1} + u_t$
- $\pi_t = \xi\theta\pi_{t-1} + (1-\xi)E_t\pi_{t+1} + \kappa x_t + e_t$
- Implication Ha: The feedback rule implies output gap negatively related to current inflation and a discounted sum of future expected inflation (which is solved out).
- Implication Hb: This feedback rule affects the convergence of inflation to its targeted level.